B.教育部補助大專校院延攬國際頂尖人才執行績效報告

(本報告將公告於計畫網站，請謹慎檢視內容是否適合對外公開)

一、基本資料

計畫核定年度	108 年		
報告年度	108 年		
學校名稱及聘任系所	國立臺灣師範大學數學系	學門領域	理學
玉山(青年)學者姓名	孟悟理	職稱	教授
聘任方式	□玉山學者 □專任教師(含編制內專任教師及編制外專案教師) □短期交流 ■玉山青年學者		
經費執行期間	108 年 10 月 9 日 至 110 年 4 月 8 日		
聯絡人	單位：國立臺灣師範大學數學系 職稱及姓名：李小慧助理 聯絡電話：02-7749-6622 傳　　真：02-2933-2342 電子信箱：cathy106@ntnu.eud.tw		

二、執行情形

(一)玉山(青年)學者工作項目及內容(如教學工作或研究計畫等)

Teaching

2018 – 2019 *Real analysis I (lecture with tutorial, joint with Chun-Chi Lin)*

Topics own part: Measures and measurable sets, Borel sets, measurable functions (approximation theorems, spaces of measurable functions), Lebesgue integration (basic properties, limit theorems).

2019 *Real analysis II (lecture with tutorial)*

Topics: Lebesgue spaces, Jensen's inequality, Daniell integrals, linear functionals on Lebesgue spaces, Riesz's representation theorem, Fubini's theorem, Lebesgue measure.

2019 *Special topics in analysis (lecture)*

Topics: Borel and Suslin sets, symmetric algebra of a vectorspace, polynomial functions, classical, approximate, and pointwise differentiation of higher order, rectifiability of higher order.

2019 – 2020 *Topics in Geometric Analysis I (MSc/PhD) (seminar, joint with Chun-Chi Lin)*

Topics: Sobolev functions and functions of bounded variation, topological vector spaces, pointwise differentiability of higher order for sets, Kohn's example concerning approximate differentiation.

2019 – 2020 *Geometric Measure Theory I (MSc/PhD) (lecture)*

Topics: Grassmann algebra (tensor producs, graded algebras, exterior algebra of a vector space, interior multiplications, simple *m*-vectors, inner products), Borel regular measures, covering theorems, derivatives of measures.

- The course is offered both at the National Taiwan Normal University and in the programme of the Taiwan Mathematics School
- Video-recordings of the lectures were made available to the participants of the course.

2020 *Topics in Geometric Analysis II (MSc/PhD) (seminar, joint with Chun-Chi Lin)*

Topics: rectifiability and approximate differentiability of higher order for sets, pointwise differentiability of higher order for distributions, Hall's theorem on perfect matching.

2020 *Geometric Measure Theory II (MSc/PhD) (lecture)*

Topics: Carathéodory's construction, curves of finite length, differentials and tangents, second fundamental form, area of Lipschitzian maps.

- The course is offered both at the National Taiwan Normal University and in the programme of the Taiwan Mathematics School.
- Video-recordings of the lectures were made available to the participants of the course.

2020 – 2021 *Topics in Geometric Measure Theory I (MSc/PhD) (seminar)*

Topics: locally convex spaces, Daniell integrals (decomposition and weak convergence), Grassmann manifolds, some structure theory, some elliptic PDEs, curvature of submanifolds, basic properties of varifolds.

2020 – 2021 *Topics in Geometric Measure Theory II (MSc/PhD) (seminar)*

Topics: distribution theory, some elliptic PDEs, first variation of area, radial deformations and the rectifiability theorem, compactness theorem for integral varifolds, isoperimetric inequality, Allard's regularity theorem.

PhD students

- Hsin-Chuang Chou, since 2019
- Yu-Tong Liu, since 2021

MSc student

- Yu-Tong Liu, 2019 - 2020

Research Summary

Generally speaking, my research aims at the understanding of the complex local structure of surfaces occurring in many models from the natural sciences. In this regard, a mathematical surface may correspond to a variety of different physical objects: for instance, soap films, horizons of black holes, membranes of cells, and boundaries between different phases of a material, or between different grey levels in a digitally reconstructed image.

一般而言，Menne 教授的研究目標在於了解局部的複雜曲面結構，它出現在許多自然科學的模型。在這方面，數學曲面可能和許多各式各樣的物理物件有關連：例如，肥皂泡膜、黑洞視界、細胞膜以及物質不同相的界面，或者經數位化重建的影像中不同灰階之間的界面。

It is the power of mathematical abstraction, that allows to devise a model of surfaces capable of covering all these cases at once and to derive theoretical conclusions (e.g., regularity results), that typically are applicable in all of these settings. By a regularity result, one means a mathematical theorem, that says, that a surface satisfying a given optimality condition admits a simpler (i.e., more regular) local description, than an arbitrary surface in its class.

正是數學抽象化的力量，讓我們設想出一個曲面模型，適用剛剛提到的所有實例，並從理論上得出結果(例如:regularity 結果)，足以代表性地應用於全部情境。所謂 regularity 結果，指的是一則數學定理，它說，當曲面滿足某個最佳條件時，可以給出更簡單（也就是更 regular）的局部描述，在同類型的曲面中，這個特性並非任意可見。

The study of models of surfaces with a complex local structure pertains to the field of geometric measure theory. The success of this theory does not only stem from the many applications, that its results have found in other larger fields within mathematics (e.g., differential geometry, geometric analysis, and mathematical models in the natural sciences), but also from the new methods that it has contributed to the big fields of partial differential equations and the calculus of variations.

具有局部複雜結構的曲面模型之研究與幾何測度論領域有關。幾何測度論的成功，不僅因為它的結果有許多應用，出現在其他更寬廣的數學領域（例如：微分幾何、幾何分析以及自然科學中的數學模型），也因為它的方法新穎，為偏微分方程與變分學這個重大領域作出貢獻。

The core motivation for my research is to make progress towards a fundamental regularity question formulated by Allard in 1972. This question concerns the local structure of surfaces in a particularly versatile class of surfaces (namely, integral varifolds) under the natural optimality condition for this class. This class of surfaces is employed in models of all of the above-mentioned physical objects.

Menne 教授的研究的核心動機在於推動基礎的 regularity 問題的進展，這個問題是 Allard 於 1972 年所提出的。這個問題關心某一類用途特別廣的曲面（也就是可積 varifolds）在符合自然最佳條件時所具有的局部結構。這類曲面用於前面提到的所有物理物件的模型之中。

My research programme in this direction consists of three, partially overlapping stages. In the first two stages, I systematically investigate which properties of their regular counterparts (namely, twice continuously differentiable submanifolds) are shared by integral varifolds in case they satisfy the optimality condition. More precisely, in the first stage (which is now completed), I studied regularity theorems for these integral varifolds, and in the second stage (which is nearing completion), I transfer mathematical machinery previously only available for the regular counterparts to the relevant classes of varifolds. In contrast, the third stage, which is currently in the starting phase, aims at establishing regularity properties that exceed those of their regular counterparts.

Menne 教授在這方面的研究計畫包含了三個不同但部分重疊的階段。在最初兩個階段，Menne 教授系統性地探究可積 varifolds 在符合最佳條件時，和他們的 regular 同類（也就是二階連續可微子流形）有哪些共同的特性。更精確地說，在第一階段（目前已完成），Menne 教授研究了這些可積 varifolds 的 regularity 定理。而在第二階段（目前已接近完成），Menne 教授把之前只適用於 regular 同類的數學機制推廣到相關類型的 varifolds 之上。往另一個方向發展，是目前剛起步的第三階段的目標，試圖建立超越他們的 regular 同類所具備的 regularity 性質。

Conference Talks

10 Nov, 2018	National Kaohsiung Normal University, Kaohsiung, Taiwan, "The Seventeenth Taiwan Geometry Symposium", "The moving plane method for generalized submanifolds with bounded mean curvature and density bounded from below uniformly", invited by Professor Chih-Wei Chan, Professor River Chiang, Professor Chung-I Ho, Professor Nan-Kuo Ho, Professor Yng-Ing Lee, and Professor Mao-Pei Tsui.
28 Jun, 2019	National Center for Theoretical Sciences, Taiwan, "International Workshop on Geometric Analysis and Harmonic Analysis", "Pointwise differentiability of higher order for distributions", invited by Professor Chun-Yen Shen.
29 Jun, 2019	National Taiwan Normal University, Taiwan, Joint forum NTNU and Kyushu University: Session Mathematics, "Pointwise differentiability of higher order for functions and sets", invited by Professor Chun-Chi Lin.
03 Nov, 2019	National Center of Theoretical Sciences, "The second Taiwan-Japan Joint Conference on Differential Geometry", "A priori geodesic diameter bounds for solutions to a variety of Plateau problems", invited by Professor Mao-Pei Tsui.
23 Jul, 2020	University of Leipzig, Germany, "Federer's 100^{th} birthday web conference", "Pointwise differentiability of higher order for distributions", invited by Dr Jonas Hirsch
06 Dec, 2020	East China Normal University, P. R. China, Workshop, "Weekend in Geometric Measure Theory and its Applications", invited by Professors Thierry De Pauw, Fanghua Lin, and Dong Ye.

Seminar Talks

21 Dec, 2018	National Taiwan Normal University, Taiwan, "TGIF! Cross-Domain Afternoon Tea", "A mathematical language for soap bubbles", invited by Professor Kwunmin Chen.
29 Jan, 2019	University of Salzburg, Austria, "Forschungsseminar Partielle Differentialgleichungen", "Pointwise differentiability of higher order for distributions", invited by Professor Verena Bögelein.
17 Oct, 2019	Academia Sinica, Taipeh, Taiwan, "Colloquium", "The concept of varifold", invited by Professor Yi-Chiuan Chen.
17 Oct, 2019	National Center of Theoretical Sciences, "NCTS Seminar on Differential Geometry", "Regularity of and geometric analysis on varifolds with mean curvature", invited by Professor Yng-Ing Lee, Professor Chung-Jun Tsai, and Professor Mao-Pei Tsui.
04 Dec, 2019	Tokyo Institute of Technology, Japan, "Mathematical analysis seminar", "A priori diameter bounds for solutions to a variety of Plateau problems", invited by Professor Yoshihiro Tonegawa.
15 Oct, 2020	Okinawa Insitute for Science and Technology, Japan, "FALL 2020 Nonlinear Analysis Seminar Series", "A priori geodesic diameter bounds for solutions to a variety of Plateau problems", invited by Professor Daniel Spector.

Recent Publications

U. Menne

Pointwise differentiability of higher order for distributions, 32 pages.

Anal. PDE, 14(2):323–354, 2021.

DOI: 10.2140/apde.2021.14.323. ArXiv: 1803.10855v2 [math.FA].

Abstract: For distributions, we build a theory of higher order pointwise differentiability comprising, for order zero, Łojasiewicz's notion of point value. Results include Borel regularity of differentials, higher order rectifiability of the associated jets, a Rademacher-Stepanov type differentiability theorem, and a Lusin type approximation. A substantial part of this development is new also for zeroth order. Moreover, we establish a Poincaré inequality involving the natural norms of negative order of differentiability. As a corollary, we characterise pointwise differentiability in terms of point values of distributional partial derivatives.

U. Menne, M. Santilli

A geometric second-order-rectifiable stratification for closed subsets of Euclidean space, 14 pages.

Ann. Sc. Norm. Super. Pisa Cl. Sci. (5), 19(3):1185-1198, 2019.

DOI: 10.2422/2036-2145.201703_021. ArXiv: 1703.09561v2 [math.CA].

Abstract: Defining the m-th stratum of a closed subset of an n dimensional Euclidean space to consist of those points, where it can be touched by a ball from at least $n-m$ linearly independent directions, we establish that the m-th stratum is second-order rectifiable of dimension m and a Borel set. This was known for convex sets, but is new even for sets of positive reach. The result is based on a sufficient condition of parametric type for second-order rectifiability

U. Menne

Pointwise differentiability of higher order for sets, 31 pages.

Ann. Global Anal. Geom., 55(3), 591–621, 2019.

Springer Nature SharedIt: https://rdcu.be/bgUqa

DOI: 10.1007/s10455-018-9642-0. ArXiv: 1603.08587v2 [math.DG].

Abstract: The present paper develops two concepts of pointwise differentiability of higher order for arbitrary subsets of Euclidean space defined by comparing their distance functions to those of smooth submanifolds. Results include that differentials are Borel functions, higher order rectifiability of the set of differentiability points, and a Rademacher result. One concept is characterised by a limit procedure involving inhomogeneously dilated sets. The original motivation to formulate the concepts stems from studying the support of stationary integral varifolds. In particular, strong pointwise differentiability of every positive integer order is shown at almost all points of the intersection of the support with a given plane.

Funding and Fellowships

2019 – 2022	Ministry of Science and Technology, Taiwan (R.O.C.) (Grant No. 108-2115-M-003-016-MY3).
2018 – 2023	Yushan Young Scholarship.

Research Visits

2019	Poland, twelve days: University of Warsaw; host Sławomir Kolasiński.

Mini-courses

2019	Seven video lectures (joint with Mario Santilli), Relating curvatures of geometric measure and convex geometry, Summer School Geometric Measure Theory and Related Fields, Beijing, China.

Organization of Research Seminars and Conferences

2019 -	Co-Organiser (with Yng-Ing Lee, Chung-Jun Tsai, and Mao-Pei Tsui), research seminar NCTS Differential Geometry Seminar; National Center for Theoretical Sciences.
2020 -	Co-Organiser (with Guido De Philippis, Yoshihiro Tonegawa, and Neshan Wickramasekera), research seminar NCTS international Geometric Measure Theory seminar with complete virtual venue; National Center for Theoretical Sciences.

Other Visiting Scholars and Speakers Invited

02 - 15 June, 2019	Brian Krummel, Purdue University, USA.
10 June, 2020	Heiner Olbermann, Catholic University of Louvain, Belgium. (Online colloquium talk funded by MoST)

Current Collaborators

* Sławomir Kolasiński, University of Warsaw, Poland,
(completed, Decay rates for the quadratic and super-quadratic tilt-excess of integral varifolds; ongoing: TBA).

* Mario Santilli, Augsburg University, Germany,
(completed, A geometric second-order-rectifiable stratification for closed subsets of Euclidean space; ongoing: TBA).

* Christian Scharrer, University of Warwick, UK,
(completed, An isoperimetric inequality for diffused surfaces; ongoing: A novel type of Sobolev-Poincaré inequality for submanifolds of Euclidean space).

(二)玉山學者團隊合作情形(請敘明團隊成員及合作方式)(玉山青年學者免填)

(三)績效說明(請說明達到量化或質化之具體成果與績效、對學校發展之具體助益等)

1. **推動國際化並促進國際合作:** Menne 教授曾任職於德國馬克斯普朗克研究院科學數學研究所 (Max Planck Institute for Mathematics in the Sciences)、阿爾伯特·愛因斯坦研究院(Albert Einstein Institute)等國際一流學術研究機構，也曾任教德國萊比錫大學、瑞士蘇黎世大學等國際名校，擁有豐富的領導學術研究團隊經驗，同時也舉辦過極高水準的國際研討會，與國際重要學者密切的互動聯繫。對於本校未來在國際合作的推動上，將扮演十分關鍵的角色。
2. **推廣全球在地化：**在產學合作層面，本校(系)正積極發展科學計算學程，其中很重要的「影像處理」部分，極度依賴於幾何測度論的知識與工具。Menne 教授不僅能支援這方面所需的研究、教學人力，也將讓本校(系)成為台灣在這個方向上位居領導的地位。所帶來的效益，不僅止於純學術方面，同時在 AI 時代來臨之背景下，這將會帶動產學合作的積極效應。
3. **豐富英文化數學課程：**Menne 教授的加入同時能滿足本校**國際教師文憑學程 - 英文化數學課程**的需要，豐富本校(系)英語授課的環境。
4. **深化本系及外校學生學術專業：**Menne 教授所涉及的研究領域可謂是數學裡的艱深研究，其認真嚴謹之教學態度在其授課課程的學生中得到一致的贊同，吸引台大、交大學生選修其教授的課程。Menne 教授積極地與學生對話並回覆學生所提出之問題來教授其專業，並配合教學錄影方式提供學生可以再次複習的機會，致力強化與深化學生之學術專業，為學生打下堅實的基礎，進而發展進階水準的數學研究。
5. **提升本校國際能見度及協助學生為邁向國際社會做準備：**通過邀請國外一流大學之學者及合作辦理國際研討會以提升本校在國際的能見度。學生善用學校裡學到的專業知識以融合國際社會，積極接軌世界，提前學習語言且了解文化的差異為國際化(go international)做好準備，同時培養學生對任何當地文化環境都可以適應的能力，Menne 教授的加入，無疑是創造三贏(國家、學校、學生)的局面。

6. **因應疫情之影響，首先舉辦線上國際研討會**：2020 年新冠肺炎疫情嚴重影響全球人口流動，造成國際間學術交流的阻礙，面對這樣的難題，Menne 教授設想了一個由外部講師組成的有關幾何測度論進展的虛擬呈現活動。本活動並邀請**美國紐約大學柯朗數學研究所** Guido De Philippis 教授、**日本東京工業大學** Yoshihiro Tonegawa 教授、**英國劍橋大學** Neshan Wickramasekera 教授與本校 Menne 教授共同籌劃，由國家理論科學研究中心協助辦理。且在此虛擬活動中，每位研究者可以免費註冊參加，因而獲得進入虛擬設施的管道，其中包括大廳、演講廳和配有白板/黑板的獨立討論會議室。因此，它將重新創造在國際會議上發現的交流可能性；對於註冊參加者，他們可以隨時使用討論室及其公告欄。活動延續促進國際學術交流的精神，提供本國學者與研究人員一個彈性靈活的管道來與世界頂尖學者進行研究交流的機會，不為疫情所困，更讓世界看見臺灣的學術研究如何精進地發展。

   詳情請參閱 https://sites.google.com/ncts.ntu.edu.tw/international-gmt-seminar 。